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On the dependence on the magnetic field orientation of the composite fermion effective mass

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Abstract. The composite fermion (CF) model has been strikingly successful in describing many aspects of the fractional quantum Hall effect (FQHE) observed in two-dimensional electron systems (2DES). In the CF picture, the FQHE is the integer quantum Hall effect of the CFs. In order to assess the effect of an in-plane magnetic field on the CFs we have examined the temperature dependence (40 mK $\leqslant T \leqslant$ 1 K) of the oscillations in ρ_{xx} in a high-mobility GaAs–(Ga, Al)As heterojunction close to Landau level filling factors $\nu = \frac{1}{2}$ and $\frac{3}{2}$ for many different values of θ, the angle between the normal to the 2DES and the magnetic field. The CF energy gaps were evaluated at each angle using a variant of the Lifshitz–Kosevich approach. Close to $\nu = \frac{1}{2}$, it was found that the CF gaps at each angle could be fitted to within experimental errors using a constant CF effective mass. However, the CF effective mass was found not to follow the θ-dependence expected for a purely 2D system; i.e. the CF energy gap at fixed ν grows markedly with increasing in-plane field. Around $\nu = \frac{3}{2}$ the situation is more complex, and the oscillations of the energy gaps at $\nu = \frac{8}{5}$, $\frac{7}{5}$ and $\frac{4}{3}$ as θ varied were interpreted using a recent model of two independent CF Landau fans separated by the Pauli spin splitting (Du R R, Yeh A S, Stormer H L, Tsui D C, Pfeiffer L N and West K W 1995 *Phys. Rev. Lett.* **75** 3926). However, whilst the model qualitatively predicts some of the behaviour of the ρ_{xx}-minima, it is unable to account for the absolute sizes of the energy gaps. In order to reproduce the gaps at $\nu = \frac{7}{5}$ and $\frac{4}{3}$ quantitatively, an angle-dependent CF mass (as observed close to $\nu = \frac{1}{2}$) is required. The data suggest that the compression of the electronic wave function due to the in-plane field and exchange effects both play a role in determining the size of the CF gaps and cast doubts on the supposed 'universal' behaviour of the CF mass.

1. Introduction

The observation of the fractional quantum Hall effect in the high-mobility two-dimensional electron system (2DES) in GaAs–(Ga, Al)As heterojunctions has generated considerable experimental and theoretical interest [1]. One of the most interesting developments in this field came with the introduction of the composite fermion approach [2–4], which exploits a gauge freedom applicable only in two dimensions. An even number $2p$ of flux quanta are attached to each electron, making a composite fermion which obeys Fermi–Dirac statistics. Owing to the attachment of the flux quanta, the composite fermions experience zero effective magnetic field at $B_{1/2p} = N_s h/e\nu$, where the magnetic field B is assumed to be perpendicular to the 2DES, N_s is the areal carrier density and $\nu = 1/2p$ is the Landau level filling factor. The fractional quantum Hall effect is then envisaged as the integer quantum Hall effect of

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the composite fermions moving in the effective field $B^* = B - B_{1/2p}$. Thus, the oscillations in the transverse component of the resistivity ρ_{xx} associated with the fractional quantum Hall effect have been analysed in the same way as conventional Shubnikov–de Haas oscillations to yield a composite fermion effective mass [5, 6]. This effective mass can be used to predict the fractional-quantum-Hall-effect gaps in a consistent manner and has been found to have a slow dependence on the total effective field [5, 6].

Although a number of experimental probes have now been used to study the properties of composite fermions (see e.g. references [5–7]), almost all work has been performed with the magnetic field applied perpendicular to the 2DES layer. In order to assess the effect of an in-plane magnetic field on the composite fermions, we have examined the temperature dependence of the oscillations in ρ_{xx} in a high-mobility GaAs–(Ga, Al)As heterojunction close to filling factors $\nu = \frac{1}{2}$ and $\nu = \frac{3}{2}$ for many different values of θ, the angle between the normal to the 2DES and the magnetic field. Close to $\nu = \frac{1}{2}$, we find that the composite fermion effective mass does not follow the angle dependence expected for a two-dimensional entity, whilst around $\nu = \frac{3}{2}$ we find that the recent model proposed by Du *et al* [8] is unable to account for the absolute values of the composite fermion energy gaps.

This paper is organized as follows. Experimental details are given in section 2, and section 3 describes the evaluation of the composite fermion energy gaps close to $\nu = \frac{1}{2}$ as a function of the tilt angle. In displaying the effect of an in-plane field, our method is to derive an average composite fermion effective mass which describes the energy gaps for $\frac{3}{5} \geqslant \nu \geqslant \frac{3}{7}$ to a reasonable accuracy and to then evaluate how this parameter varies with tilt angle. Section 4 covers the evaluation of energy gaps close to $\nu = \frac{3}{2}$ as a function of tilt angle and section 5 gives a comparison of their sizes with a modification of the model of reference [8]. A summary is given in section 6.

2. Experimental details

The sample used was a 500 μm wide Hall bar of accurately known geometry constructed on a GaAs–(Ga, Al)As heterojunction with an undoped spacer layer width of 120 nm. The wafer was mounted next to a Ru_2O_3 temperature sensor with a well known magnetoresistance correction in the plastic tail of an air-spring-damped top-loading dilution refrigerator. Both sample and thermometer were immersed in the ^{3}He–^{4}He mixture and cooled slowly ($\sim$10 hours) from room temperature to 50 mK in zero magnetic field. After brief illumination in zero magnetic field with a red LED at 50 mK, the sample had the areal carrier density $N_s = 1.2 \times 10^{15}$ m^{-2} and electron mobility $\mu = 900$ m^2 V^{-1} s^{-1}. The sample was rotated *in situ* so that the surface normal made angles $\theta = 0.0$, 18.3, 28.2, 39.1, 43.4, 48.9, 54.1, 60.0 and 70.6° to the magnetic field. At each angle, simultaneous measurements of ρ_{xx} and ρ_{xy} were performed at around twelve temperatures in the range 40 mK $\leqslant T \leqslant$ 900 mK, each temperature being stabilized electronically. In addition, ρ_{xx} and ρ_{xy} were recorded for many more tilt angles at a constant temperature of 140 mK. Standard low-frequency ($\sim$18 Hz) lock-in techniques were employed for the magnetoresistance measurements, using an A.C. current of 10 nA; the direction of current flow was always kept perpendicular to the applied magnetic field (i.e. the Hall bar was rotated about its long axis). Great care was taken to avoid sample heating due either to the applied current or to external sources of noise. The sweep rate of the superconducting magnet was always $\leqslant$0.5 T min^{-1}, and any hysteresis effects in the cycling of the superconducting magnet were kept to a minimum by recording ρ_{xx}- and ρ_{xy}-data only on upsweeps.

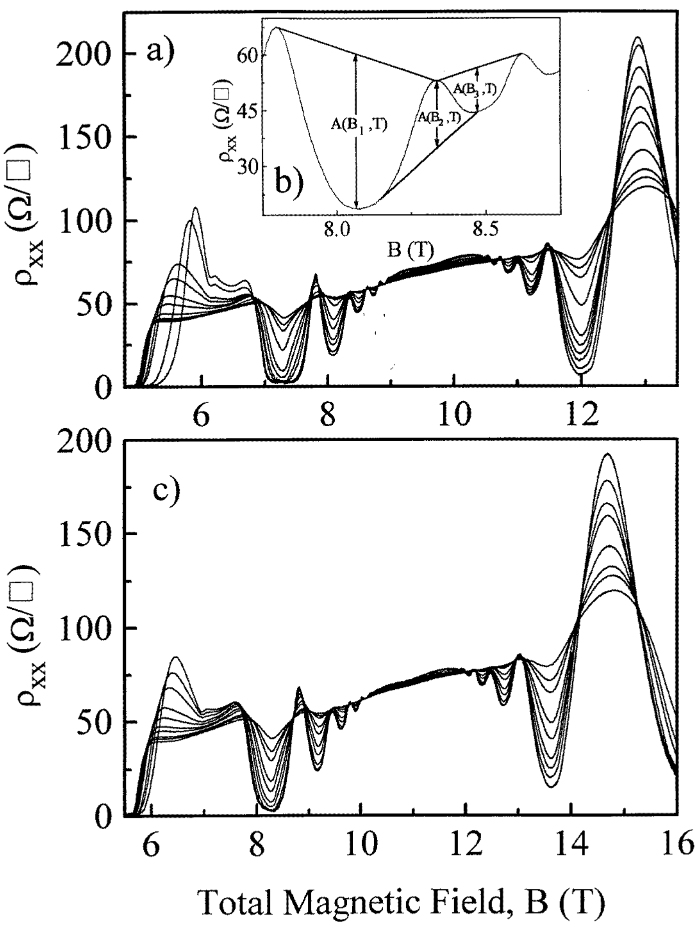

Figure 1. (a) shows ρ_{xx}-data close to $\nu = \frac{1}{2}$ (at 9.75 T) for temperatures $40 \leqslant T \leqslant 850$ mK with the magnetic field applied perpendicular to the 2DES ($\theta = 0$). (b) illustrates the definition of the oscillation amplitudes used in equation (2). (c) shows ρ_{xx}-data for $\theta = 28.2°$ and $100 \leqslant T \leqslant 850$ mK.

3. Experimental results close to $\nu = \frac{1}{2}$

Figure 1(a) shows ρ_{xx} for several different temperatures close to filling factor $\nu = \frac{1}{2}$; the magnetic field is perpendicular to the 2DES ($\theta = 0$). The minima in ρ_{xx} which will be used in the analysis below are visible at fractional filling factors $\frac{3}{5}$, $\frac{4}{7}$, $\frac{5}{9}$, $\frac{6}{11}$, $\frac{7}{13}$, $\frac{6}{13}$, $\frac{5}{11}$, $\frac{4}{9}$ and $\frac{3}{7}$, corresponding to integer composite fermion filling factors ν^*, where

$$\nu = \frac{\nu^*}{2p\nu^* \pm 1} \tag{1}$$

and p is equal to one for this series of composite fermion states [5, 6]. Furthermore, the maxima in ρ_{xx} between the above-mentioned fractions (i.e. at half-integer ν^*) are also used in the following analysis.

The procedure for extracting the composite fermion effective mass $m_{\rm cf}$ starts by assuming that the oscillations in ρ_{xx} on either side of $\nu = \frac{1}{2}$ are the Shubnikov–de Haas

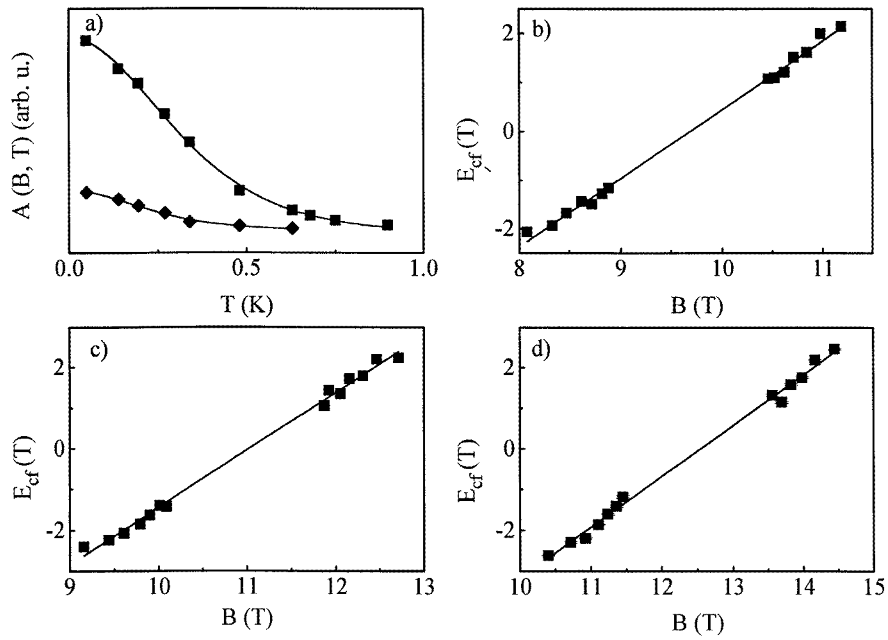

Figure 2. (a) contains fits of equation (2) to the amplitude of the ρ_{xx}-oscillations at $\nu = \frac{3}{5}$ (squares) and $\nu = \frac{5}{9}$ (diamonds). (b), (c) and (d) show the energy gaps E_{cf} versus total field B at angles $\theta = 0$, 28.2° and 39.1° respectively; points are experimental data and lines are straight-line fits. The energy gaps are plotted in units of $\hbar e/m_e$ (i.e. teslas) as the gradients of the straight-line fits then give directly the reciprocal of the composite fermion effective mass in units of m_e.

oscillations of the composite fermions at low effective field [5, 6]. The analysis of the data is based on the techniques applied to the Shubnikov–de Haas and de Haas–van Alphen oscillations of metals [9]; the amplitude $A(B^*, T)$ of each oscillation in ρ_{xx} (see figure 1(b); note that both maxima and minima in ρ_{xx} are used) is fitted to the temperature-dependent phase-smearing term of the Lifshitz–Kosevich formula [9]:

$$A(B^*, T) = F(B^*)\frac{\chi}{\sinh(\chi)} \tag{2}$$

where $F(B^*)$ is a function of only the effective magnetic field [9], $\chi = 2\pi^2 k_B T/E_{cf}$, $E_{cf} = \hbar e B^*/m_{cf}$ is the separation of the composite fermion Landau levels, and m_{cf} is the composite fermion effective mass. Typical fits of equation (2) to the data are shown in figure 2(a). As the Lifshitz–Kosevich formalism can only be used for small, approximately sinusoidal oscillations in ρ_{xx}, the analysis in this paper was restricted to the fractions listed above [10]; lower-numerator fractions (e.g. $\nu = \frac{2}{3}, \frac{2}{5}$) are too large for such methods over the temperature range used [11].

As has already been mentioned, our method of displaying the effect of an in-plane field is to derive an average composite fermion effective mass which describes the energy gaps for $\frac{3}{5} \geqslant \nu \geqslant \frac{3}{7}$ to a reasonable accuracy [11] and to then evaluate how this parameter varies with tilt angle. The procedure for obtaining the average composite fermion mass is displayed in figure 2(b), which shows the values of E_{cf} plotted as a function of *total field* (the sign of E_{cf} has been reversed for negative B^*); the points lie on a straight line passing through $E_{\rm cf} \approx 0$ at $\nu = \frac{1}{2}$. This result shows that to the limit of experimental accuracy the composite fermion levels are symmetrical about $\nu = \frac{1}{2}$ and that $m_{\rm cf}$ does not vary greatly over this small range of effective field [5, 6]. Hence all of the fractional quantum Hall gaps studied are described with reasonable accuracy using one composite fermion effective mass [13], and an average value of $m_{\rm cf}$ can be extracted from the gradient of the straight line in figure 2(b).

In order to check that the results did not depend on the sample contact geometry, $E_{\rm cf}$ was also extracted by fitting equation (2) to the amplitude of the oscillations in $\mathrm{d}\rho_{xy}/\mathrm{d}B$ [15] obtained by numerically differentiating the Hall data; within the experimental errors, the same values of $E_{\rm cf}$ were obtained [16].

Figure 1(c) shows ρ_{xx} close to $\nu = \frac{1}{2}$ at a tilt angle of $\theta = 28.2°$. Resistivity features observed at magnetic fields $B(0)$ at $\theta = 0$ have shifted up to higher fields $B(\theta) = B(0)/\cos\theta$, as expected for a 2DES. As has been mentioned above, in the CF picture in perpendicular field, $E_{\rm cf} = \hbar e B^*/m_{\rm cf}$. As the functional form of the oscillations close to $\nu = \frac{1}{2}$ does not appear to change drastically with tilt angle, we assume that $E_{\rm cf}$ also has the form $\hbar e(B - B_{1/2})/\mu$ at other field orientations, where B and $B_{1/2}$ are the *total* magnetic fields at which corresponding features are observed and $\mu(\theta)$ is a mass to be determined.

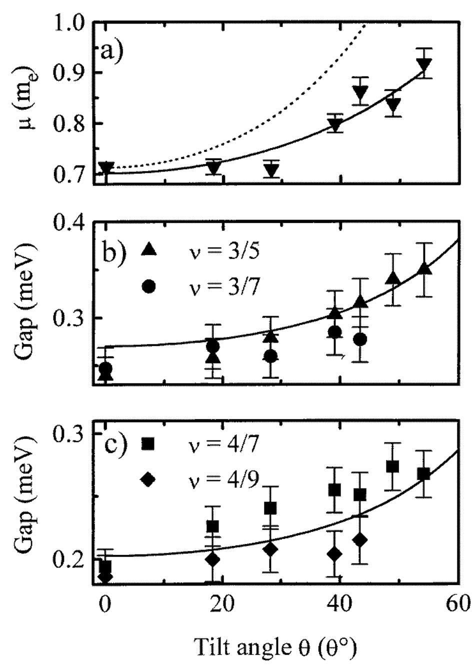

Figure 3. (a) shows values of the average composite fermion mass μ derived from straight line fits such as those in figures 2(b)–2(d) versus θ (inverted triangles). The dotted line depicts the function $\mu = m_{\rm cf}/\cos\theta$ and the solid line depicts equation (3). (b) shows the energy gaps for $\nu = \frac{3}{5}$ and $\frac{3}{7}$ as a function of θ generated using $E = \hbar e|(B - B_{1/2})|/\mu(\theta)$ with $\mu(\theta)$ from equation (3) (solid curve). (c) shows a similar calculation for $\nu = \frac{4}{7}$ and $\frac{4}{9}$ (solid curve). For comparison, experimental values for the gaps are also shown (triangles, $\frac{3}{5}$; circles, $\frac{3}{7}$; squares, $\frac{4}{7}$; diamonds, $\frac{4}{9}$); the reasonable agreement between calculation and data shows that $\mu(\theta)$ is a suitable means for describing the variation of all gaps observed with θ.

Averaged values of μ were determined by finding E_{cf} using equation (2) and then plotting the values as a function of total field B; figures 2(c) and 2(d) show examples of this for $\theta = 28.2^\circ$ and 39.1°. Using all of the available data, a plot of μ versus θ can then be made, and this is shown as figure 3(a). If the composite fermions were purely 2D entities, one would expect E_{cf} to depend only on the perpendicular component of the magnetic field, $B\cos\theta$. In this case $E_{cf} = \hbar e(B - B_{1/2})\cos\theta/m_{cf}$, yielding $\mu = m_{cf}/\cos\theta$. Figure 3(a), however, shows that μ increases more slowly with θ than $1/\cos\theta$; in fact the variation of μ with θ may be fitted empirically by the function

$$\mu(\theta) = \frac{0.70m_e}{\cos(0.75\theta)} \qquad (3)$$

where m_e is the free-electron mass. No theoretical significance can be attached to this equation; it is merely a convenient way of parametrizing the variation of μ which will be useful in later analysis.

Equation (3) implies that the composite fermion energy gaps are growing with increasing tilt angle, perhaps due to the increasing component of magnetic field in the plane of the 2DES. To emphasize this point, equation (3) has been used to generate the magnitudes of the E_{cf} at various fractional ν as a function of θ in figures 3(b) and 3(c); if the composite fermions were purely two dimensional, the gap would depend only on the component of B perpendicular to the 2DES ($B\cos\theta$) and so would be a constant. We shall return to this point in section 5 once data around $\nu = \frac{3}{2}$ have been discussed.

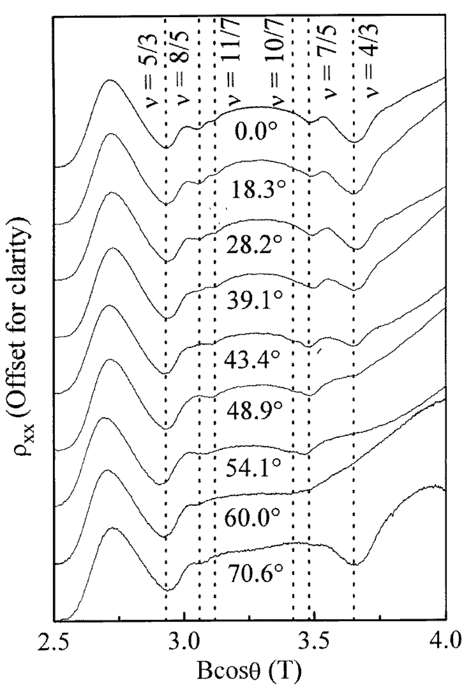

Figure 4. Plots of ρ_{xx} around $\nu = \frac{3}{2}$ for several values of the tilt angle θ at $T = 140$ mK; the field axis depicts the component of B perpendicular to the 2DES. The vertical dotted lines show the positions of fractional filling factors ν.

4. Experimental results close to $\nu = \frac{3}{2}$

Figure 4 shows ρ_{xx} close to $\nu = \frac{3}{2}$ at several tilt angles θ. In the composite fermion picture, the fractional-quantum-Hall-effect states at around $\nu = \frac{3}{2}$ are regarded as the $\nu' = 2 - \nu$

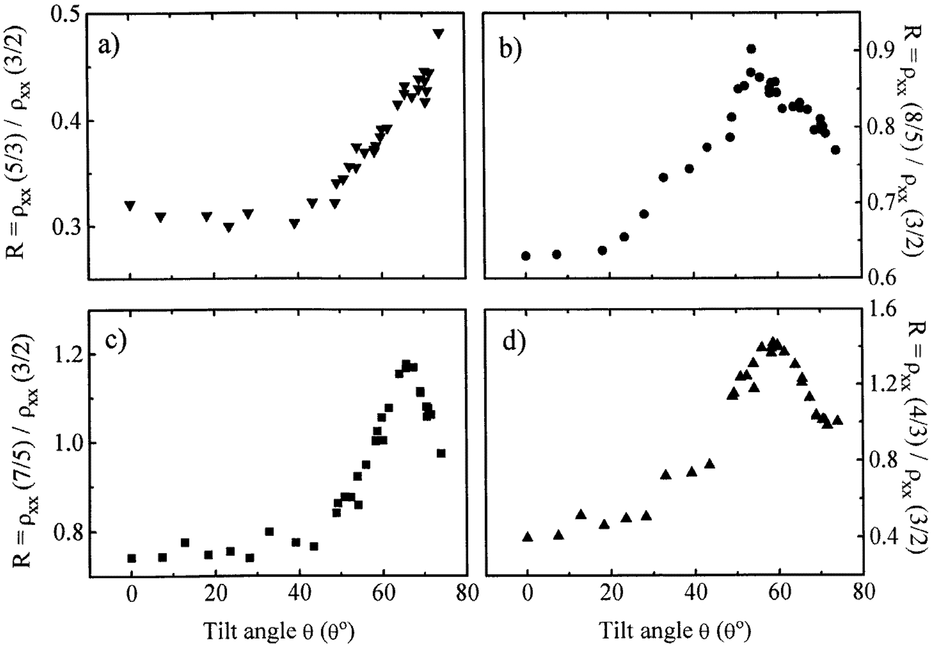

Figure 5. $R(r/s) = \rho_{xx}(\nu = r/s)/\rho_{xx}(\nu = \frac{3}{2})$ versus θ for $r/s = \frac{5}{3}$ (a), $\frac{8}{5}$ (b), $\frac{7}{5}$ (c) and $\frac{4}{3}$ (d). Maxima indicate angles at which the energy gap falls to a very small value.

states made up of holes in the upper spin state of the lowest electron Landau level [8]; the fractional quantum Hall states become equivalent to composite fermion Landau levels due to an effective field $B^* = 3(B - B_{3/2})$ [8, 17]. As has been noted on a number of previous occasions [8, 18, 19], the minimum at $\nu = \frac{5}{3}$ remains strong, whereas those at $\nu = \frac{8}{5}, \frac{11}{7}, \frac{10}{7}, \frac{7}{5}$ and $\frac{4}{3}$ oscillate in strength as θ is varied. Figures 5(a), 5(b), 5(c), and 5(d) emphasize this fact by displaying $R(r/s) = \rho_{xx}(\nu = r/s)/\rho_{xx}(\nu = \frac{3}{2})$ as a function of θ at a constant temperature of 140 mK (the normalization is carried out to remove any background θ-dependence of the resistivity [8]). In the cases of $\nu = r/s = \frac{8}{5}, \frac{7}{5}$ and $\frac{4}{3}$, $R(r/s)$ exhibits maxima at the angles $53 \pm 3°$, $65 \pm 2°$ and $59 \pm 2°$ respectively, indicating that the energy gap at $\nu = r/s$ has collapsed to a small value at these points. However, $R(\frac{5}{3})$ indicates no strong peak as a function of angle.

Energy gaps at $\nu = \frac{5}{3}, \frac{8}{5}, \frac{7}{5}$ and $\frac{4}{3}$ were derived using the temperature dependence (40 mK–1 K) of the oscillations in ρ_{xx} in a similar fashion to that detailed in section 3. In some cases, the strong temperature dependence of the background resistivity or the close proximity of other fractions meant that the amplitude of the oscillation in ρ_{xx} was difficult to define. When this occurred, the value of ρ_{xx} at the fractional-quantum-Hall-effect minimum was fitted to a modified Lifshitz–Kosevich formula:

$$\rho_{xx}(B^*, T) = C - F(B^*)\frac{\chi}{\sinh(\chi)} \tag{4}$$

(cf. equation (2)) where C is a parameter (allowed to vary in the fitting procedure) representing the background magnetoresistance from which the oscillations deviate. This method was checked against the more conventional analysis (equation (2)) for the well defined oscillations and was found to give the same results within the experimental errors.

This indicates the validity of the method, and in particular that the parameter C does not, for example, depend strongly on the temperature.

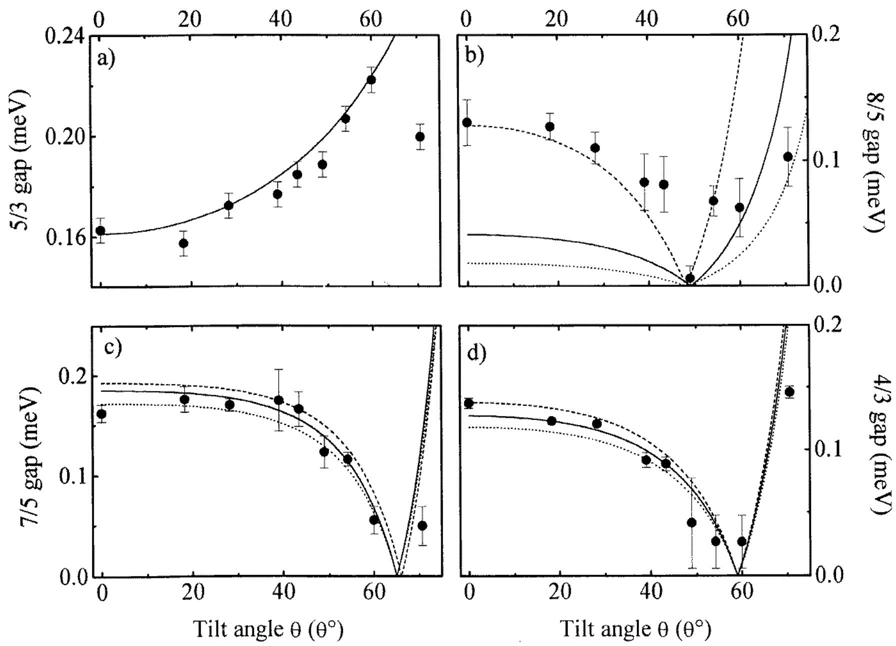

Figure 6. Energy gaps for ρ_{xx}-minima at around $\nu = \frac{3}{2}$ versus the tilt angle θ. (a) The gap for $\nu = \frac{5}{3}$. Points are data and the curve is equation (8). (b) The gap for $\nu = \frac{8}{5}$. Points are data and the curves are generated by equations (9) and (10) with $\mu'(0) = 0.1m_e$, $g_{\rm eff} = 3.2$ (dashed line); $\mu'(0) = 0.33m_e$, $g_{\rm eff} = 0.91$ (solid line); $\mu'(0) = 0.7m_e$, $g_{\rm eff} = 0.46$ (dotted line). (c) The gap for $\nu = \frac{7}{5}$. Points are data and the curves are generated by equations (9) and (10) with $\mu'(0) = 0.325m_e$, $g_{\rm eff} = 0.997$ (dashed line); $\mu'(0) = 0.325m_e$, $g_{\rm eff} = 1.01$ (solid line); $\mu'(0) = 0.35m_e$, $g_{\rm eff} = 0.972$ (dotted line). (d) The gap for $\nu = \frac{4}{3}$. Points are data and the curves are generated by equations (9) and (10) with $\mu'(0) = 0.30m_e$, $g_{\rm eff} = 1.57$ (dashed line); $\mu'(0) = 0.325m_e$, $g_{\rm eff} = 1.45$ (solid line); $\mu'(0) = 0.35m_e$, $g_{\rm eff} = 1.35$ (dotted line).

Energy gaps at $\nu = \frac{5}{3}, \frac{8}{5}, \frac{7}{5}$ and $\frac{4}{3}$ derived in this manner are shown as functions of θ in figure 6. Note that the energy gaps at $\frac{8}{5}, \frac{7}{5}$ and $\frac{4}{3}$ decrease to very small values at angles close to those at which $R(r/s)$ exhibited maxima (see figures 5(a)–5(d)).

5. Discussion

In interpreting the data displayed in the two previous sections, two phenomena must be taken into account, the non-two-dimensional nature of the composite fermion effective mass and the vanishing of some of the composite fermion gaps at around $\nu = \frac{3}{2}$ at particular angles. Noting the similarity of the latter phenomenon to the vanishing of the spin splitting of conventional Shubnikov–de Haas oscillations in two-dimensional systems at certain tilt angles (i.e. the so-called 'coincidence' [20] or 'harmonic ratio' [21] methods to determine the g-factor) Du *et al* have proposed an ingenious interpretation of the latter phenomenon [8]. In this approach, the composite fermion Landau levels close to $\nu = \frac{3}{2}$ are assumed to be split by a Pauli spin-splitting term $\pm\frac{1}{2}g_{\rm eff}\mu_B B$ (here $g_{\rm eff}$ is the effective g-factor and μ_B is the Bohr magneton) which depends on the total magnetic field. However, Du *et al* proposed that the composite fermion cyclotron energy depends only on the component of the effective field perpendicular to the 2DES, $(B - B_{3/2})\cos\theta$. Thus the composite fermion energy levels are assumed to have the form [8]

$$E(n) = -\frac{\hbar e}{m_e}\left\{\frac{3|(B - B_{3/2})|\cos\theta(n + \frac{1}{2})}{(m_{cf}/m_e)} \pm \frac{g_{eff}B}{4}\right\} \tag{5}$$

where $n = 0$, 1, 2, 3 etc. and the minus sign takes account of the fact that these are 'hole-like' levels. The factor 3 is included to give the correct value of the effective field around $\nu = \frac{3}{2}$ [8, 17] and m_{cf} is assumed (in the approach of Du *et al*) to be independent of θ. The filling factors ν at which minima in ρ_{xx} are observed are then related to the composite fermion filling factors ν^* (i.e. the number of filled composite fermion hole levels) by [8]

$$\nu = \frac{3\nu^* \pm 2}{2\nu^* \pm 1}. \tag{6}$$

Equation (5) predicts that if one remains at a constant value of ν or ν^* (i.e. at a constant value of $B\cos\theta$), the effect of tilting the magnetic field is to increase the spin splitting relative to the cyclotron splitting; this results in the energy gaps (i.e. composite fermion level separations) at particular filling factors vanishing at certain angles θ_v. Combining equations (5) and (6), the values of θ_v at which the gaps at $\nu = \frac{8}{5}$, $\frac{7}{5}$ and $\frac{4}{3}$ first vanish are given by

$$\cos\theta_v = g_{eff}\frac{m_{cf}}{m_e}f \tag{7}$$

where $f = 2.5$, 1.25 and 1.5 for $\nu = \frac{8}{5}$, $\frac{7}{5}$ and $\frac{4}{3}$ respectively. These θ_v may be identified with angles $53\pm3^\circ$, $65\pm2^\circ$ and $59\pm2^\circ$ at which $R(r/s)$ exhibits maxima for $\nu = r/s = \frac{8}{5}$, $\frac{7}{5}$ and $\frac{4}{3}$ respectively (figures 5(a)–5(d)). Substituting these angles into equation (7) yields $g_{eff}m_{cf}/m_e = 0.24 \pm 0.02$, 0.34 ± 0.02 and 0.34 ± 0.02 for $\nu = \frac{8}{5}$, $\frac{7}{5}$ and $\frac{4}{3}$ respectively. Du *et al* performed a similar analysis and obtained similar values for $g_{eff}m_{cf}$ [8], tentatively attributing the increase in the value $g_{eff}m_{cf}$ as the filling factor decreased to increasing exchange enhancement of g_{eff} (for a discussion of such effects see e.g. reference [20]).

Although the model of reference [8] provides an appealing qualitative solution to the vanishing of various fractional quantum Hall features near $\nu = \frac{3}{2}$ at certain angles, it cannot give a satisfactory quantitative account of the sizes of the gaps shown in figure 6. Attempts to use the predictions of equations (5) and (6) with a fixed, angle-independent m_{cf} and a g_{eff} varied to fit the angles at which the gaps vanish were unsuccessful in the cases $\nu = \frac{8}{5}$, $\frac{7}{5}$ and $\frac{4}{3}$. However, perhaps the most marked failure concerns the gap associated with $\nu = \frac{5}{3}$; the use of the values of m_{cf} and g_{eff} given in reference [8] with equation (5) predicts that this gap is determined solely by the spin splitting at low tilt angles. It should therefore grow as $1/\cos\theta$ [8]. In figure 6(a) this is seen not to be the case, as the gap increases much more slowly than $1/\cos\theta$ up to $\theta \sim 65^\circ$ and then decreases. A clue as to what might be happening is given by comparing the θ-dependence of the gap at $\nu = \frac{5}{3}$ (figure 6(a)) with those of the gaps close to $\nu = \frac{1}{2}$ (figures 3(b) and 3(c)); the functional form of all of the curves is very similar. Indeed, the gap at $\nu = \frac{5}{3}$ is fitted quite well by the function

$$3\hbar e|(B - B_{3/2})|/\mu \tag{8}$$

with μ taking the values given by equation (3) (see figure 6(a)). This suggests that at angles $\theta \leqslant 65^\circ$ the energy gap at $\nu = \frac{5}{3}$ has a similar origin to those of the fractions around $\nu = \frac{1}{2}$.

In an attempt to fit the variation of the gaps at $\nu = \frac{8}{5}$, $\frac{7}{5}$ and $\frac{4}{3}$, equation (5) was modified to the form

$$E(n) = -\frac{\hbar e}{m_e}\left\{\frac{3|(B - B_{3/2})|(n + \frac{1}{2})}{(\mu'/m_e)} \pm \frac{g_{\text{eff}}B}{4}\right\} \tag{9}$$

with

$$\mu' = \mu'(0)/\cos(0.75\theta). \tag{10}$$

That is, the composite fermion effective mass was allowed to vary with θ in a similar fashion to that observed close to $\nu = \frac{1}{2}$ (equation (3)), but with $\mu'(0)$ as a fitting parameter.

Typical fits are shown as curves in figures 6(b)–6(d) and the fit parameters are detailed in the figure captions. Successful fits were obtained for $\nu = \frac{7}{5}$ and $\frac{4}{3}$ (figures 6(c) and 6(d)) with fit parameters $\mu'(0) = 0.33 \pm 0.03m_e$ (both $\nu = \frac{7}{5}$ and $\frac{4}{3}$), and $g_{\text{eff}} = 1.50 \pm 0.05$ ($\nu = \frac{7}{5}$) and $g_{\text{eff}} = 1.45 \pm 0.10$ ($\nu = \frac{4}{3}$). The values of $\mu'(0)$ are close to the values of composite fermion effective masses at around $\nu = \frac{3}{2}$ proposed and justified in reference [8]. Whilst the values of g_{eff} are a factor $\sim$3 times the g-factor of bulk GaAs, such enhancements due to exchange interactions are by no means large nor without precedent [20].

In contrast, although some aspects of the functional form of the energy gap associated with $\nu = \frac{8}{5}$ were predicted by equations (9) and (10), it proved impossible to fit the variation quantitatively using sensible parameters (see figure 6(b) and the caption). Considering also the much larger effective mass ($0.7m_e$) used to fit the gap at $\nu = \frac{5}{3}$ (figure 6(a)) compared to those used with $\nu = \frac{4}{3}$ and $\frac{7}{5}$ ($0.33m_e$; figures 6(c) and 6(d)) it seems possible that the behaviours of the composite fermion energy levels in the cases where $\nu \leqslant \frac{3}{2}$ and $\nu \geqslant \frac{3}{2}$ are different.

6. Summary and conclusions

The results described in section 3 suggest that the fractional quantum Hall effect close to $\nu = \frac{1}{2}$ in tilted magnetic fields may still be described in a consistent manner by the composite fermion model, but with a composite fermion effective mass that depends on the tilt angle θ. It seems that the component of field in the plane of the 2DES causes the energy gaps to increase in an approximately uniform fashion (figures 3(b) and 3(c)). At present, there appears to be little theoretical work on the dynamics of composite fermions in tilted magnetic fields. However, in the period before the composite fermion approach evolved, some work was carried out on the effect of an in-plane field on the fractional quantum Hall effect (see e.g. reference [22]). In such works the fractional-quantum-Hall-effect gaps were predicted to increase due to the compression of the electronic wave function in the direction perpendicular to the plane of the 2DES caused by the in-plane component of the magnetic field; i.e. the in-plane component of the field makes the electrons appear more ideally two dimensional. Such an effect may well be causing the enhancement of the gaps close to $\nu = \frac{1}{2}$ seen in the present work. If this is the case, then the effect should depend on the exact shape of the 2DES potential; e.g. an in-plane field should have far less effect on a 2DES in a narrow quantum well than in a heterojunction. Studies of the composite fermion effective mass in a perpendicular field have suggested that the mass is some ‘universal’ function of the effective magnetic field (although the exact form of the function is still contentious) [5, 6]. However, all of these experiments have been carried out on GaAs–(Ga, Al)As heterojunctions with very low depletion charges and remote dopants, in which the potential well shapes will be rather similar. If the angle dependence of the composite fermion effective mass observed in the present work is due to the compressing effect of the in-plane magnetic field, it suggests that the 'universal' behaviour of the composite fermion mass in a perpendicular field may well disappear when different material systems or potential well shapes are studied.

Turning to the results close to $\nu = \frac{3}{2}$, it seems that the ingenious composite fermion model proposed by Du *et al* [8] can qualitatively account for some of the disappearing fractional ρ_{xx}-minima, but cannot predict energy gaps quantitatively. With the inclusion of an angle-dependent effective mass, quantitative values of the gaps for fractions with $\nu \leqslant \frac{3}{2}$ can be predicted. However, the gaps for $\nu \geqslant \frac{3}{2}$ cannot be fitted using similar parameters. It seems that whilst the composite fermion approach provides an elegant description of the rough form of the ρ_{xx}-oscillations near $\nu = \frac{3}{2}$, some other ingredient (e.g. a very strongly field-dependent g-factor) is as yet missing.

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Note added in proof. In section 6 of this paper it was proposed that the orientation dependence of the composite fermion (CF) effective mass resulted from the compressing effect of the in-plane component of the magnetic field. During the production of this paper, we were able to demonstrate that this assertion is in fact correct using a model based on the Fang–Howard variational wavefunction and the calculations in reference [22]. Our model successfully reproduces the angle dependence of the experimental CF masses with no adjustable parameters and is described in reference [23].

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